Hello everyone! It's time for summer math! It's designed to help students maintain their math skills during the summer break. Research shows that all students experience learning losses during the summer when they do not engage in educational activities. On average, students lose approximately 2.6 months of grade level equivalency in mathematical computations during the summer months (Harvard Graduate School of Education).

For extra motivation, students that complete it and bring it back will earn extra credit! Remember, this is not mandatory, but this is a great resource to keep students sharp for the next school year and it's a great opportunity for bonus points next school year! The deadline to turn it in is August 8, 2024, the first day of school. Make the most of this opportunity and let's start the new school year strong!

Fractions

(Addition, Subtraction, Multiplication, and Division)

Miscellaneous

Write the fractions in lowest terms.

1. $\frac{8}{24}$

2. $\frac{18}{24}$

3. $\frac{15x^2y}{20xy}$

4. $\frac{36abc^4}{45a^3bc^2}$

Solve for x.

5. $\frac{16}{48} = \frac{x}{12}$

6. $\frac{12}{42} = \frac{4}{x}$

7. $\frac{20}{32} = \frac{x}{16}$

8. $\frac{6}{9} = \frac{12}{x}$

Write as improper fractions.

9. $2\frac{1}{3}$

10. $-4\frac{6}{7}$

Write as mixed numbers.

11. $-\frac{9}{4}$

12. $\frac{38}{3}$

Addition and Subtraction

Find each sum or difference. Write your answer in simplest form.

13. $-\frac{2}{3}+\frac{1}{4}$

14. $3\frac{5}{9}+2\frac{1}{6}$

15. $\frac{3}{10}-\frac{4}{5}$

16. $6\frac{7}{10}+\left(-1\frac{1}{5}\right)$

17. $5\frac{4}{11}-2\frac{2}{3}$

18. $2\frac{7}{12}-9\frac{2}{3}$

Multiplication and Division

Find each product or quotient. Write your answer in simplest form.

19. $-\frac{5}{6}\cdot\frac{6}{15}$

20. $-\frac{3}{4}\div\left(-\frac{9}{16}\right)$

21. $2\frac{2}{5}\cdot\left(-3\frac{3}{4}\right)$

22. $-3\frac{3}{4}\div 4\frac{2}{3}$

23. $\frac{2}{9}\cdot\frac{3}{16}\cdot\frac{3}{6}$

24. $6\frac{3}{4}\div 4$

Order of Operations

When several operations are indicated in a numerical expression, proceed in the following order: work within the parentheses, expand each power, multiply and divide (whichever comes first), and finally, add or subtract (whichever comes first).

PEMDAS ("**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally") is an acronym that provides a good way to remember your order of operation.

- **P**: Parentheses
- **E**: Exponents
- **MD**: Multiply or Divide, whichever comes first
- **AS**: Add or Subtract, whichever comes first

Simplify.

1. $2^4 - 3(3^2 - 8)$

2. $(4^2 + 10)4 - 10(5^2 - 20)$

3. $4^2 - 4(5^2 - 32 \div 8 \cdot 4)$

4. $(8 \cdot 5 \div 10 + 2)(2^5 - 8^2 \div 2)$

5. $5^2 - 3\left[6 + (-2)\left(20 + (-15)\right)\right]$

6. $[4^3 + (-10)(30 - 8 \cdot 5)]$

7. $[15 - 3(4^2 - 10) + 25 \div 5 \cdot 15]$

8. $\{10 - 5[20 - 2(3^2 + 1)]\}$

9. $|-32| + 32$

10. $\frac{48-24\div 2^3}{3+2\cdot 6}$

Working with Integers

Adding and Subtracting:

1st: Rewrite all subtraction as addition then…

- If the integers have the same signs, add their absolute values. The sum will have the same sign of the addends.
- If the integers have different signs, subtract their absolute values. The sum has the sign of the addend with the greater absolute value.

Multiplying and Dividing:

- The product or quotient of two integers having the same sign is positive.
- The product or quotient of two integers having different signs is negative.

Find each sum, difference, product, or quotient.

1. $-13 + 19$

2. $37 + (-13)$

3. $-18 + (-29)$

4. $-27 - 93$

5. $-46 - (-32)$

6. $9 - 83$

7. $-45 \div 9$

8. $-84 \div -12$

9. $\frac{132}{-11}$

10. $8(-17)$

11. $-24 \cdot -6$

12. $-62(8)$

13. There is a 6° drop in temperature over the past hour. If it is 55° now, what was the temperature an hour ago?

14. It is −9° now. The temperature will drop 5° in two hours. What will the temperature be in two hours?

Evaluating Algebraic Expressions

1. Substitute the given values for the variables in the expression
2. Evaluate the expression using the order of operations
   - Parentheses/Brackets (inside to outside)
   - Exponents
   - Multiplication/Division (left to right)
   - Addition/Subtraction (left to right)

ex: $9x^2 - 4(y + 3z)$
for $x = -3, y = 2, z = 5$

$9(-3)^2 - 4(2 + 3 \cdot 5)$

$9(-3)^2 - 4(2 + 15)$

$9(-3)^2 - 4 \cdot 17$

$9 \cdot 9 - 4 \cdot 17$

$81 - 4 \cdot 17$

$81 - 68 = \boxed{13}$

The Distributive Property

1. Multiply the number outside the parentheses by each term in the parentheses.
2. Keep the addition/subtraction sign between each term.

ex: $5(8x - 3)$

$5(8x - 3)$

$5(8x) - 5(3)$

$\boxed{40x - 15}$

Simplifying Algebraic Expressions

1. Clear any parentheses using the Distributive Property
2. Add or subtract like terms (use the sign in front of each term to determine whether to add or subtract)

ex: $2(3x - 4) - 12x + 9$

$2(3x - 4) - 12x + 9$

$6x - 8 - 12x + 9$

$\boxed{-6x + 1}$

Evaluate each expression for $a = 9$, $b = -3$, $c = -2$, $d = 7$. Show your work.

1. $a - cd$	2. $2b^3 + c^2$	3. $\frac{a + d - c}{b}$	4. $(a - b)^2 + d(a + c)$
5. $4c - (b - a)$	6. $\frac{a}{b} - 5a$	7. $2bc + d(12 - 5)$	8. $b + 0.5[8 - (2c + a)]$

Simplify each expression using the Distributive Property.

9. $5(2g - 8)$	10. $7(y + 3)$	11. $-3(4w - 3)$	12. $(6r + 3)2$

Simplify each expression, showing all work.

13. $8(x + 1) - 12x$	14. $6w - 7 + 12w - 3z$	15. $9n - 8 + 3(2n - 11)$	16. $3(7x + 4y) - 2(2x + y)$
17. $(15 + 8d)(-5) - 24d + d$	18. $9(b - 1) - c + 3b + c$	19. $20f - 4(5f + 4) + 16$	20. $8(h - 4) - h - (h + 7)$

Solving One-Step Equations

1. Cancel out the number on the same side of the equal sign as the variable using inverse operations (addition/subtraction; multiplication/division)
2. Be sure to do the same thing to both sides of the equation!

ex: $-18 = 6j$

$\frac{-18}{6} = \frac{6j}{6}$

$-3 = j \rightarrow \boxed{j = -3}$

Solving Two-Step Equations

1. Undo operations one at a time with inverse operations, using the order of operations in reverse (i.e. undo addition/subtraction before multiplication/division)
2. Be sure to always do the same thing to both sides of the equation!

ex: $\frac{a}{7} - 12 = -9$

$\frac{a}{7} - 12 = -9$

$+12 \quad +12$

$\frac{a}{7} = 3 \times 7$

$\boxed{a = 21}$

Solving Multi-Step Equations

1. Clear any parentheses using the Distributive Property
2. Combine like terms on each side of the equal sign
3. Get the variable terms on the same side of the equation by adding/subtracting a variable term to/from both sides of the equation to cancel it out on one side
4. The equation is now a two-step equation, so finish solving it as described above

ex: $5(2x - 1) = 3x + 4x - 1$

$10x - 5 = 3x + 4x - 1$

$10x - 5 = 7x - 1$

$-7x \quad -7x$

$3x - 5 = -1$

$+5 \quad +5$

$\frac{3x}{3} = \frac{4}{3}$

$\boxed{x = \frac{4}{3}}$

Solve each equation, showing all work.

21. $f - 64 = -23$	22. $-7 = 2d$	23. $\frac{b}{-12} = -6$	24. $13 = m + 21$
25. $5x - 3 = -28$	26. $\frac{w + 8}{-3} = -9$	27. $-8 + \frac{h}{4} = 13$	28. $22 = 6y + 7$
29. $8x - 4 = 3x + 1$	30. $-2(5d - 8) = 20$	31. $7r + 21 = 49r$	32. $-9g - 3 = -3(3g + 2)$
33. $5(3x - 2) = 5(4x + 1)$	34. $3d - 4 + d = 8d - (-12)$	35. $f - 6 = -2f + 3(f - 2)$	36. $-2(y - 1) = 4y - (y + 2)$

Slope & Rate of Change

Finding the Slope Given Two Points: Use the coordinates from the points in the slope formula:

$$\text{Slope } (m) = \frac{y_2 - y_1}{x_2 - x_1}$$

ex: $(4, -2)$, $(-3, 8)$ — x_1 y_1 x_2 y_2

$$m = \frac{8 - (-2)}{-3 - 4} = \frac{10}{-7} = \boxed{-\frac{10}{7}}$$

Finding the Rate of Change From a Table: Determine the amount the dependent variable (y) is changing and the amount the independent variable (x) is changing.

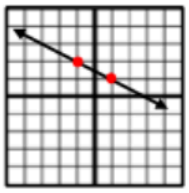
$$\text{Rate of Change} = \frac{\text{change in } y}{\text{change in } x}$$

ex:

		+2	+2	+2	
x	# months	3	5	7	9
y	Cost ($)	80	130	180	230
		+50	+50	+50	

$$m = \frac{50}{2} = \boxed{25 \text{ dollars/month}}$$

Finding the Slope From a Graph: Choose 2 points on the graph. Find the vertical change (rise) and horizontal change (run) between the 2 points and write it as a fraction $\frac{\text{rise}}{\text{run}}$. (Up is positive, down is negative, right is positive, and left is negative).



rise = +1
run = -2

$$m = \frac{1}{-2} = \boxed{-\frac{1}{2}}$$

Graphing Linear Equations

Slope-Intercept Form: $y = mx + b$ (m: slope, b: y-intercept)

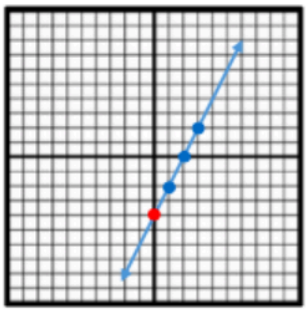
ex: $y = 2x - 4$

y-intercept: -4

slope: $2 = \frac{2}{1}$ ← rise / run

How To Graph:

1. Make a point on the y-axis at the y-intercept.
2. Use the slope to determine where to make the next point. The numerator tells you the rise (how far up/down) and the denominator tells you the run (how far right/left) to make the next point.
3. Repeat to make more points and then connect the points with a line.



Find the slope of the line that passes through the points. Show your work.

61. (-5, 3), (2, 1)	62. (8, 4), (11, 6)	63. (9, 3), (9, -1)	64. (-4, -2), (-6, 4)

Find the rate of change. Show your work.

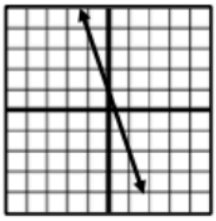
65.

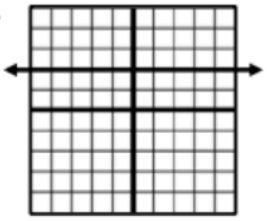
Number of Hours	3	6	9	12
Distance (in miles)	135	270	405	540

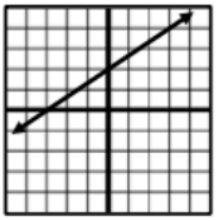
66.

Number of Weeks	1	3	5	7
Pounds	173	169	165	161

Find the slope of the line.

67. 

68. 

69. 

Graph the line.

70. $y = -x - 3$

71. $y = \frac{1}{3}x + 2$

72. $y = -3x - 1$

73. $y = -\frac{3}{2}x - 2$

74. $y = 2x + 1$

75. $y = \frac{1}{4}x$

Laws of Exponents

Hints/Guide:

There are certain rules when dealing with exponents that we can use to simplify problems. They are:

Adding powers	$a^m a^n = a^{m+n}$
Multiplying powers	$(a^m)^n = a^{mn}$
Subtracting powers	$\frac{a^m}{a^n} = a^{m-n}$
Negative powers	$a^{-n} = \frac{1}{a^n}$
To the zero power	$a^0 = 1$

Here are some examples of problems simplified using the above powers:

$4^3 \bullet 5^5 = 4^8$ $\quad (4^3)^3 = 4^9$ $\quad 4^5 \div 4^3 = 4^2$ $\quad 4^{-4} = \frac{1}{4^4} = \frac{1}{256}$ $\quad 4^0 = 1$

Exercises: Simplify the following problems using exponents (Do not multiply out).

1. $5^2 5^4 =$
2. $7^{-3} 7^5 =$
3. $(12^4)^3 =$
4. $(6^5)^2 =$
5. $5^9 \div 5^4 =$
6. $10^3 \div 10^{-5} =$
7. $7^{-3} =$
8. $3^{-4} =$
9. $124^0 =$
10. $-9^0 =$
11. $(3^5 \bullet 3^2)^3 =$
12. $5^3 \bullet 5^4 \div 5^7 =$