Are You Ready for Math Beyond Algebra!

It's time for summer math! We would like to make sure that you continue to learn this summer by completing the following algebra math topics. Research shows that all students experience learning losses during the summer when they do not engage in educational activities. On average, students lose approximately 2.6 months of grade level equivalency in mathematical computations during the summer months (Harvard Graduate School of Education). At the beginning of each topic, there are example problems to refresh your memory. If you're still stuck, I highly recommend Khan Academy as a resource.

Please turn in your completed summer math assignment to your MATH teacher on the first day of school to receive extra credit!

Wishing you a safe and relaxing summer!

Name _____ Date _____ Period _____

Are You Ready for Pre-Calculus?
Prerequisite Skills

· **Part I. Linear Equations in Two Variables:** Notes and Review

Slope Formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Equations of lines:
Vertical Line: $x = \#$ Slope of a vertical line is undefined or no slope.
Horizontal Line: $y = \#$ Slope of a horizontal line is 0.
Slope-Intercept Form: $y = mx + b$ where m is the slope (rise over run) and b is the y-intercept $(0, b)$.
Point-Slope Form: $y - y_1 = m(x - x_1)$
Standard Form (General Form): $Ax + By = C$ where $A, B,$ and C are integers (no fractions or decimals) and A is positive.
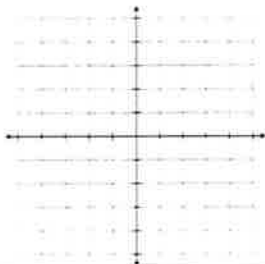
Parallel and Perpendicular Lines:
Parallel lines have the same slope.
Perpendicular lines have slopes that are negative reciprocals of each other, that is, $m_1 = \dfrac{-1}{m_2}$. The product of the slopes of two perpendicular lines is -1.

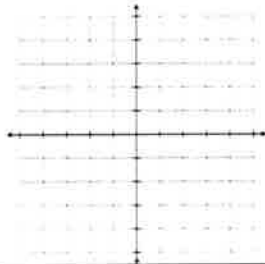*Directions:* For each linear equation, find the slope, y-intercept, and graph the line.

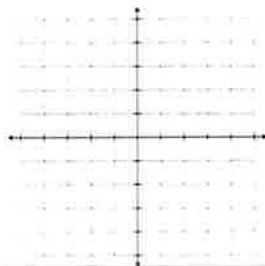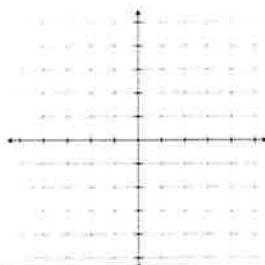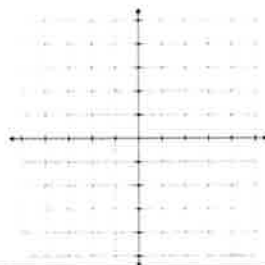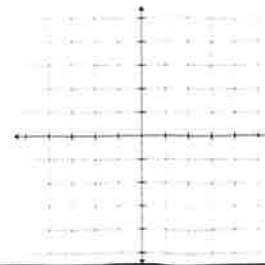|		
1. $y = \frac{1}{2}x - 4$ $m =$ $b =$	**2.** $2x - 6 = 0$ $m =$ $b =$	**3.** $y + 3 = 2$ $m =$ $b =$
4. $4x - 2y = 10$ $m =$ $b =$	**5.** $2x + 3y = 3$ $m =$ $b =$	**6.** $-2y = 5x - 4$ $m =$ $b =$
7. $4y + 3x = 0$ $m =$ $b =$	**8.** $6x - 3 = -15$ $m =$ $b =$	**9.** $-4x + 4y = 16$ $m =$ $b =$
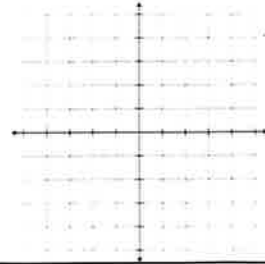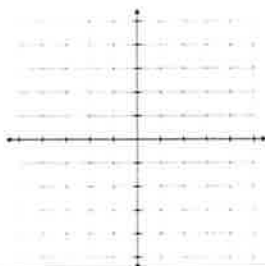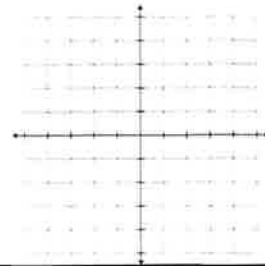
Name _____ Date _____ Period _____

Are You Ready for Pre-Calculus?
Prerequisite Skills

Directions: Read each question. Show all your work. Write your answer neatly.

1. Find the slope of the line passing through the points $(-5, 4)$ and $(7, 8)$	**2.** Find the slope of the line passing through $\left(\frac{7}{8}, \frac{1}{4}\right)$ and $\left(\frac{-5}{4}, \frac{3}{4}\right)$	**3.** Write the equation of a line passing through the point $(-3, 4)$ with a slope of ½ in point-slope form.
4. Take your equation from question #3 and write it in slope-intercept form.	**5.** Now take your equation from question #4 and write it in standard form.	**6.** Determine whether the lines passing through the following pairs of points are parallel, perpendicular, or neither: Line 1: $(-4, 6), (5, 9)$ Line 2: $(0, -½), (3, ½)$
7. Write the equation of a line with a slope of zero passing through the point $(-6, 7)$.	**8.** Write the equation of a line with undefined slope passing through the point $(-6, 7)$.	**9.** Write the equation of a line perpendicular to the line $y = ¾\, x - 2$ passing through the point $(0, -5)$ in standard form.

Factoring in a Nutshell

1. GCF (Greatest Common Factor):

First start by factoring out the GCF (greatest common factor). Ask yourself, are all of the terms divisible by the same number? Do they all have the same variable(s) in common?

Example: $24x^4 - 12x^3 + 48x^5$
What number are all of the terms divisible by? 12
What variables do they have in common? x^3
GCF = $12x^3$
Factored form: $24x^4 - 12x^3 + 48x^4 = 12x^3(2x - 1 + 4x^2)$

2. Difference of Two Squares: $a^2 - b^2 = (a+b)(a-b)$ or $a^2 - b^2 = (a-b)(a+b)$

Perfect squares are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, ... basically any positive integer that when square rooted equals an integer (no decimals). AFTER looking for the GCF (always the first step in factoring), ask yourself: is it a binomial (two terms)? Are the two terms separated by a minus sign? Are both terms perfect squares?

Example: $4x^2 - 36y^2$
Look out for any GCFs. In this case both terms are divisible by 4. When you factor out the 4, you're left with: $4(x^2 - 9y^2)$. Notice that $x^2 - 9y^2$ is a difference of two squares!
This is the factored form of $x^2 - 9y^2 = (x + 3y)(x - 3y)$. Therefore, $4x^2 - 36y^2 = 4(x + 3y)(x - 3y)$.

3. Factoring by grouping: $ax + ay + bx + by$

This technique is used when there are an even number of terms such as 4 or 6. After finding the GCF of the entire polynomial, group the first two terms together and the last two terms together. Find the GCF of each group.

Example #1: $ax + ay + bx + by = (ax + ay) + (bx + by)$. After separating the 4 terms into two groups of 2, look at the first group and factor out its GCF: $ax + ay = a(x + y)$ and look at the second group and factor out its GCF: $bx + by = b(x + y)$.
So far: $ax + ay + bx + by = a(x + y) + b(x + y)$. What do they now have in common? $x + y$
Factor it out: $(x + y)(a + b)$

Example #2: $x^3 + 2x^2 - 3x - 6$
The GCF of the first two terms is x^2 and the GCF of the last two terms is -3.
So $x^3 + 2x^2 - 3x - 6 = x^2(x + 2) - 3(x + 2) = (x^2 - 3)(x + 2)$
Notice: if the first factor would've been $x^2 - 4$, we could've continued factoring.

4. Trinomial with a leading coefficient of 1: $x^2 + bx + c$

After factoring out any GCFs, does it have three terms in the form $x^2 + bx + c$?
Look at c and ask yourself, what numbers multiply together to give me c (in other words, what are factors of c)? Then look to see what set of factors when ADDED together equal b (remember negative factors).

Example #1: $2x^2 + 28x + 80$
The GCF is 2. Factor it out to get $2(x^2 + 14x + 40)$. So now let's look at 40 and list its factors: 1x40, 2x20, 4x10, 5x8. Which set of factors add up to equal 14?
So $2x^2 + 28x + 80 = 2(x + 4)(x + 10)$.

Example #2: $x^2 - 5x - 14$

The GCF is 1 so no need to factor anything out. Factors of -14 are 1x-14, 2x-7, -1x14, -2x7. Which of those factors add to equal -5? The factors 2x-7.

Solution: $x^2 - 5x - 14 = (x + 2)(x - 7)$

5. Trinomials with a leading coefficient not equal to 1: $ax^2 + bx + c, a \neq 1$

There are several techniques to factor these types of trinomials, including trial and error. The technique explained here is going to use factoring by grouping. As always, begin by looking for a GCF for the trinomial. Then multiply a and c. Now you have to find the factors of that new number ac that when added will give you b.

So for ***example #1***: $6x^2 + 11x - 10$. Start by multiplying 6 and -10. This equals -60. What are the factors of -60: 1x-60, 2x-30, 3x-20, 4x-15, 5x-12, 6x-10, 10x-6, 12x-5, 15x-4, 20x-3, 30x-2, 60x-1. Of these possibilities, which factors add up to equal positive 11? Positive 15 times -4. So the next step is to rewrite the trinomial as a polynomial of 4 terms.

$$6x^2 + 11x - 10 = 6x^2 + 15x - 4x - 10$$

Proceed to factor by grouping.
$$= 3x(2x + 5) - 2(2x + 5)$$
$$= (3x - 2)(2x + 5)$$

Example #2: $2x^2 + 7x - 4$

Multiply 2 times -4 = -8. List the factors of -8: 1x-8, 2x-4, 4x-2, 8x-1. Which factors add up to equal positive 7? 8 and negative 1. Rewrite the trinomial so you can factor by grouping.

$$2x^2 + 7x - 4 = 2x^2 + 8x - x - 4$$
$$= 2x(x + 4) - 1(x + 4)$$
$$= (2x - 1)(x + 4)$$

6. Sum or Difference of Cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ and $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

You first need to recognize perfect cubes: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, etc.

Example #1: $y^3 - 64$

Start by rewriting it so that you can see what number cubed is 64: $y^3 - 4^3$. Now, ignoring the cubes, write down exactly what you see: $y - 4$. This is the first factor. Then you're left with the first term, y, squared. The first sign is always positive. Since here the factor is $y - 4$, the sign that follows y^2 will be plus (opposite signs). Multiply 4 and y together to get the middle term, and finally, square the last term 4 to get 16. The last sign is always plus.

$y^3 - 64 = (y - 4)(y^2 + 4y + 16)$ whereas $y^3 + 64 = (y + 4)(y^2 - 4y + 16)$.

Example #2: $125 + 8x^3 = 5^3 + 2^3x^3 = (5 + 2x)(25 - 10x + 4x^2)$

Opposite signs

Always plus

Ignore the cubes and you're left with $5 + 2x$ Where 5 is the first term and $2x$ is the second term

Square the first term: $5^2 = 25$

Multiply the two terms together. Ignore the signs. So here, 5 times $2x$ is $10x$

Square the second term: $(2x)^2 = 4x^2$

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Part II. Factoring: Notes and Review

1. <u>GCF (Greatest Common Factor)</u>
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2. <u>Difference of Two Squares:</u> $a^2 - b^2 = (a+b)(a-b)$ or $a^2 - b^2 = (a-b)(a+b)$
Perfect squares are 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, ... basically any positive integer that when square rooted equals an integer (no decimals). AFTER looking for the GCF (always the first step in factoring), ask yourself: is it a binomial (two terms)? Are the two terms separated by a minus sign? Are both terms perfect squares?

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So for ***example #1***: $6x^2 + 11x - 10$. Start by multiplying 6 and -10. This equals -60. What are the factors of -60: 1x-60, 2x-30, 3x-20, 4x-15, 5x-12, 6x-10, 10x-6, 12x-5, 15x-4, 20x-3, 30x-2, 60x-1. Of these possibilities, which factors add up to equal positive 11? Positive 15 times -4. So the next step is to rewrite the trinomial as a polynomial of 4 terms.
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Multiply 2 times -4 = -8. List the factors of -8: 1x-8, 2x-4, 4x-2, 8x-1. Which factors add up to equal positive 7? 8 and negative 1. Rewrite the trinomial so you can factor by grouping.
$$2x^2 + 7x - 4 = 2x^2 + 8x - x - 4$$
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$y^3 - 64 = (y - 4)(y^2 + 4y + 16)$ whereas $y^3 + 64 = (y + 4)(y^2 - 4y + 16)$.

Opposite signs   Always plus

Example #2: $125 + 8x^3 = 5^3 + 2^3x^3 = (5 + 2x)(25 - 10x + 4x^2)$

Ignore the cubes and you're left with $5 + 2x$ Where 5 is the first term and $2x$ is the second term

Square the first term: $5^2 = 25$

Multiply the two terms together. Ignore the signs. So here, 5 times $2x$ is $10x$

Square the second term: $(2x)^2 = 4x^2$

Name _____ Date _____ Period _____

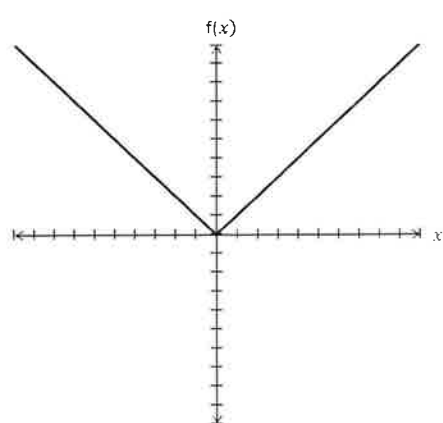**Directions:** Factor completely. Show all work where necessary.

1. $x^2 + 5x + 6$	**2.** $y^2 + 15y + 36$	**3.** $p^2 - 12p + 27$
4. $a^2 + 10a - 75$	**5.** $-x^2 - x + 20$	**6.** $x^2 - 16$
7. $2x^2 - 8$	**8.** $25x^2 - 1$	**9.** $81x^2 - 1$
10. $2x^2 - 27x + 36$	**11.** $3x^2 + 7x - 20$	**12.** $16x^2 - 48x$
13. $2y^2 - 16y + 32$	**14.** $-6a^2 - 600$	**15.** $x^2 - y^2$
16. $2p^2 - 11p + 5$	**17.** $4y^2 + 15y - 4$	**18.** $x^4 - y^4$
19. $x^3 + 2x^2 + 5x + 10$	**20.** $x^3 + 2x^2 - 3x - 6$	**21.** $x^3 - 125$
22. $8x^3 + 1$	**23.** $125x^3 - 27$	**24.** $216x^3y^3 - 343z^3$
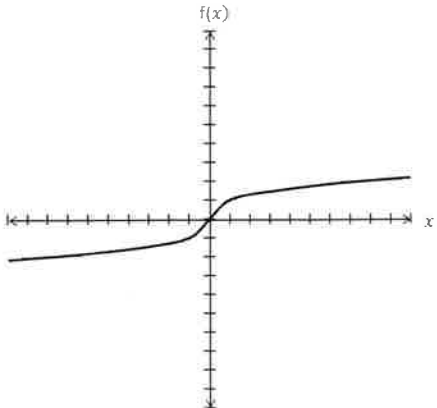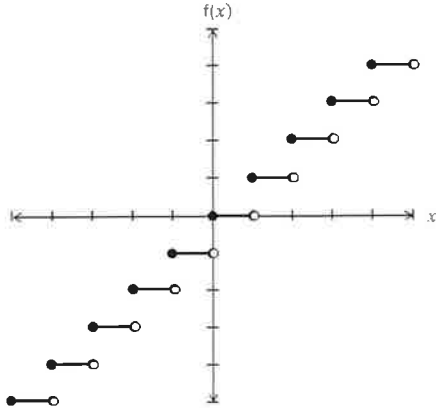
Are You Ready for Pre-Calculus?
Prerequisite Skills

Part III. Library of Functions: Notes and Review

Linear: $f(x) = x$	Quadratic: $f(x) = x^2$
D: $(-\infty, \infty)$ R: $(-\infty, \infty)$ x-int: 0 y-int: 0 slope = 1 Symmetric w/ respect to the origin (Odd) inc: $(-\infty, \infty)$	D: $(-\infty, \infty)$ R: $y > 0$ vertex: $(0, 0)$ x-int: 0 y-int: 0 Symmetric w/ respect to the y-axis (Even) dec: $(-\infty, 0)$ inc: $(0, \infty)$ abs. min at $x = 0$
Absolute Value: $f(x) = |x|$	Cubic: $f(x) = x^3$
D: $(-\infty, \infty)$ R: $y > 0$ x-int: 0 y-int: 0 Symmetric w/ respect to the y-axis (Even) dec: $(-\infty, 0)$ inc: $(0, \infty)$ abs. min at $x = 0$	D: $(-\infty, \infty)$ R: $(-\infty, \infty)$ x-int: 0 y-int: 0 Symmetric w/ respect to the origin (Odd) inc: $(-\infty, \infty)$
Square Root: $f(x) = \sqrt{x}$	Reciprocal: $f(x) = \dfrac{1}{x}$
D: $x \geq 0$ R: $y \geq 0$ x-int: 0 y-int: 0 Not even or odd inc: $(0, \infty)$ abs. min of 0 at $x = 0$	D: $x \neq 0$ R: $y \neq 0$ No x- or y-int Symmetric w/ respect to the origin (Odd) dec: $(-\infty, 0) \cup (0, \infty)$
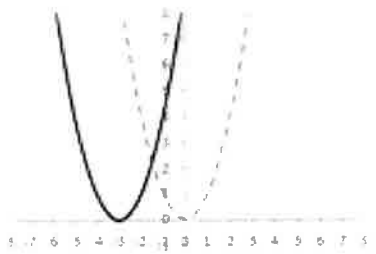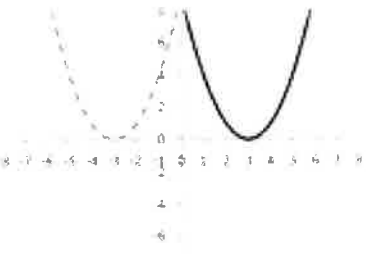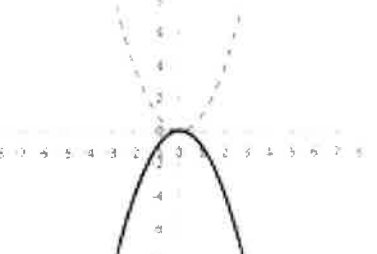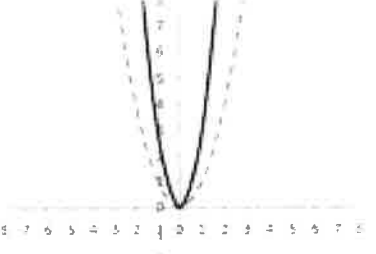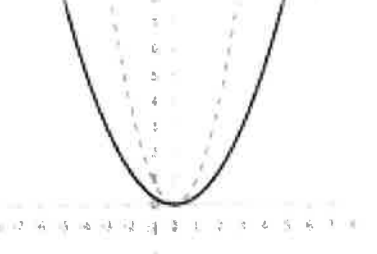
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Part III. Library of Functions: Notes and Review (cont'd)

Cube Root: $f(x) = \sqrt[3]{x}$	
$f(x)$ graph	D: $(-\infty, \infty)$ R: $(-\infty, \infty)$ x-int: 0 y-int: 0 Symmetric w/ respect to the origin (Odd) inc: $(-\infty, \infty)$

Greatest Integer (Step): $f(x) = [\![x]\!]$	
$f(x)$ graph	D: $(-\infty, \infty)$ R: $integers$ x-int: $[0,1)$ y-int: 0 Not even or odd Constant: $[k, k + 1) for$ $k\ an\ integer$ Discontinuous
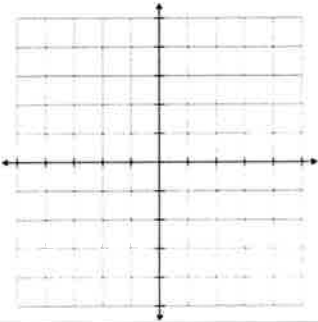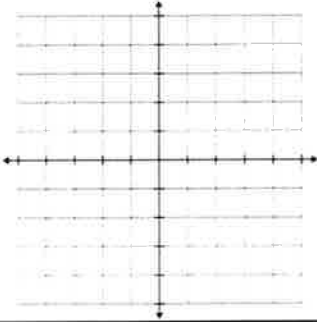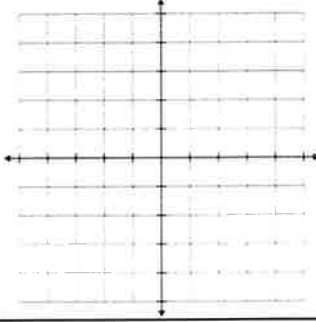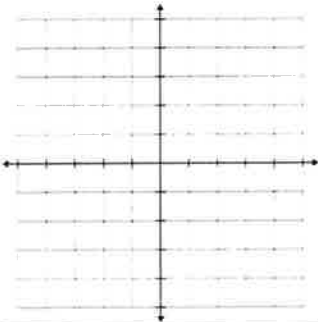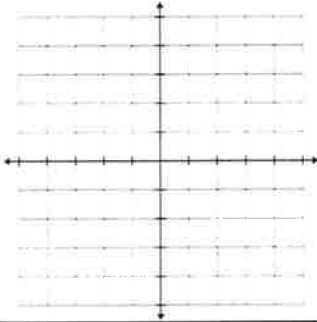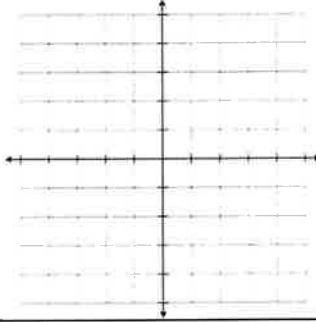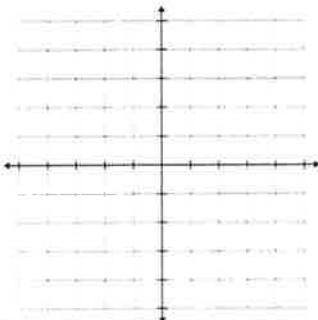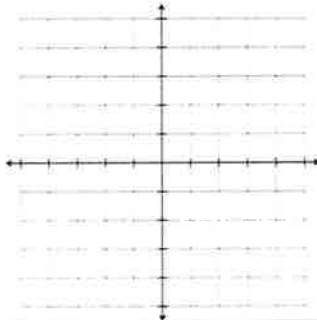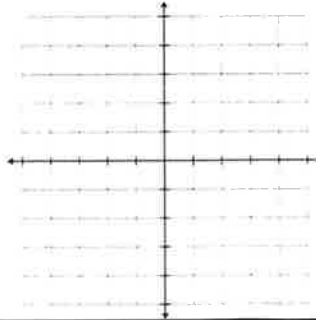
Part IV. Graphing Functions using Transformations: Notes and Review

Types of Transformations

$y = f(x) \pm k$ *vertical translation* $+$ up, $-$ down $f(x) = x^2 + 3$	$y = f(x \pm h)$ *horizontal translation* $+$ left, $-$ right $f(x) = (x + 3)^2$	$y = f(-x)$ *reflection about the $y - axis$* $f(x) = (-x + 3)^2$
graph	*graph*	*graph*
$y = -f(x)$ *reflection about the $x - axis$* $f(x) = -x^2$	$y = a\,f(x), a > 1$ *vertical stretch* *(graph gets narrower)* $f(x) = 3x^2$	$y = a\,f(x), 0 < a < 1$ *vertical compressions* *(graph gets wider)* $f(x) = \frac{1}{3}x^2$
graph	*graph*	*graph*

Name _____ Date _____ Period _____

Are You Ready for Pre-Calculus?
Prerequisite Skills

Directions: For each problem, <u>describe in words the transformation shown</u> (up 4 units, left 3 units, vertical compression, reflection about the x-axis, etc.). Then graph the function along with its parent function. Use dashed lines or different colors to distinguish between the parent function graph and the graph of the given function.

1. $f(x) = |x| - 2$	**2.** $f(x) = |x - 1| + 2$	**3.** $f(x) = \frac{1}{2}|x| - 4$
4. $f(x) = \sqrt{x + 4} - 1$	**5.** $f(x) = \sqrt{-x} - 2$	**6.** $f(x) = 2\sqrt{x - 1}$
7. $f(x) = -x^3 + 2$	**8.** $f(x) = \frac{1}{x} + 2$	**9.** $f(x) = \sqrt[3]{-x}$

Name _____ Date _____ Period _____

Are You Ready for Pre-Calculus?
Prerequisite Skills

Part V. Circles: Notes and Review

Standard form of the equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$, with its cetner at (h, k) and a radius of r.
Circle with center at the origin: $x^2 + y^2 = r^2$, with its center at the origin $(0, 0)$ and a radius of r.
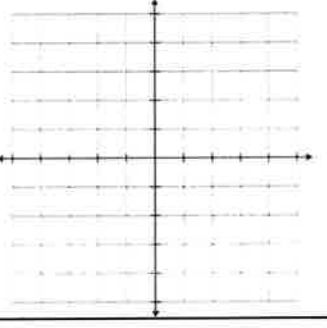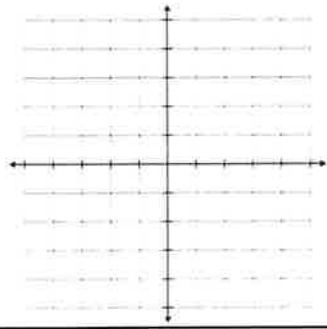Given two endpoints, find the center of the circle by using the midpoint formula: $M = (\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2})$
Given the center and one endpoint, find the radius of the circle by using the distance formula:
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
Given two endpoints, find the radius of the circle by using the distance formula to find the diameter and then dividing by 2 to find the radius (remember radius = ½ diameter)
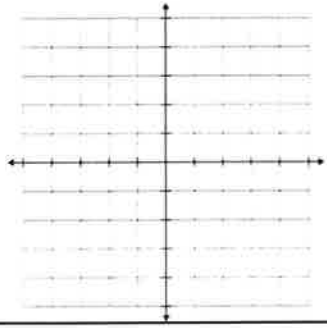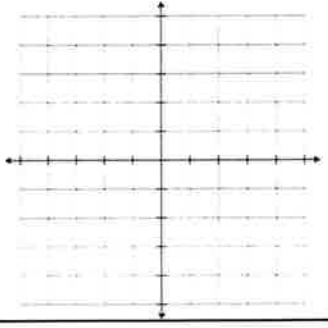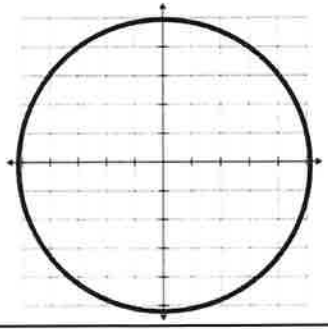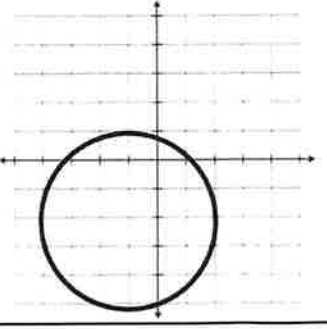
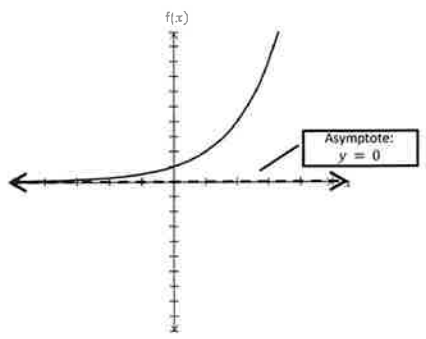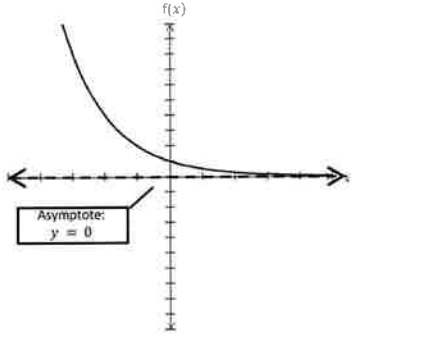**Directions:** Reach each question. Show all work where needed.

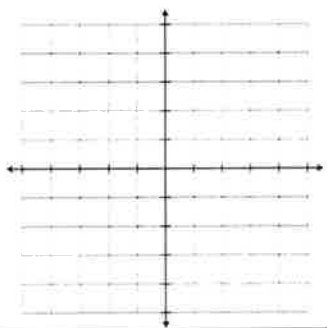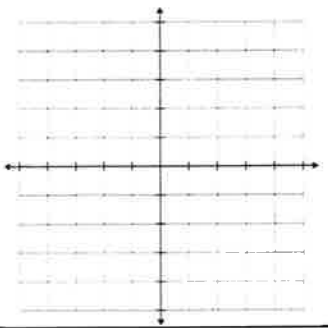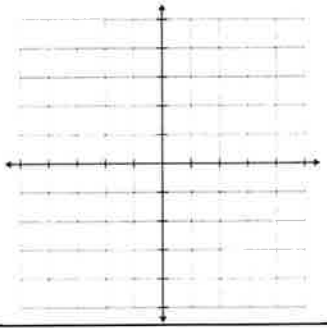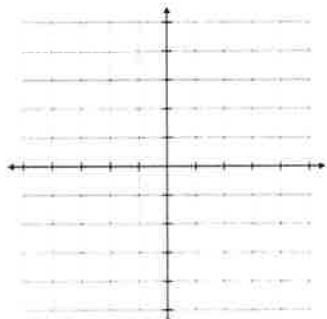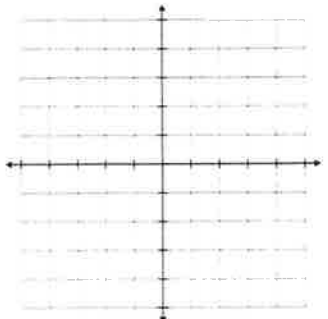|		
1. Graph the circle centered at $(-2, -1)$ with a diameter of 4. Write its equation.	**2.** Write the equation of the circle centered at the origin with a diameter of 9. Then graph the circle.	**3.** Write the equation of the circle centered at (3, 2) with a diameter of 2. Then graph the circle.
4. Find the equation of the circle with endpoints (0, 0) and (4, 6). Then graph the circle.	**5.** Look at the graph of the circle. Write its equation.	**6.** Look at the graph of the circle. Write its equation.

Are You Ready for Pre-Calculus?
Prerequisite Skills

Part V. Exponential Functions: Notes and Review

$$y = ab^x$$

If $a > 0$ and $b > 1$, the function represents <u>exponential growth.</u>	If $a > 0$ and $0 < b < 1$, the function represents <u>exponential decay</u>.
$y = 2^x$	$y = (0.5)^x$

Domain: $(-\infty, \infty)$
Range: $y > 0$
y-intercept: (0, a)
x-intercept: none
asymptote: $y = 0$
one-to-one function
Smooth
Continuous

Directions: Graph each exponential function. Make sure to include the asymptote.

1. $f(x) = 3^x + 1$

Asymptote: $y =$

2. $f(x) = (0.25)^x - 2$

Asymptote: $y =$

3. $f(x) = 2^{x+2}$

Asymptote: $y =$

4. $f(x) = -3^x + 1$

Asymptote: $y =$

5. $f(x) = 2 + 4^{x-1}$

Asymptote: $y =$

6. $f(x) = 3(0.5)^x$

Asymptote: $y =$