AP Calculus AB

Summer Assignment 2023

Welcome to AP Calculus AB! As you know, AP courses are rigorous and move quick. It is imperative that you have mastered the essential skills from Algebra and Precalculus in order to be successful in this course.

This summer assignment serves as a review of precalculus, it is chapter one in our textbook; however, we will cover this before the school year begins in order to save time. I have also included some additional skills from Algebra that you need to know.

It is expected that you have mastered the skills in this packet **before coming to class in August.** This assignment will be collected for a grade and you will have an assessment on these skills during the first week of school.

If you are unable to complete this assignment or do not pass the first assessment, we may need to rethink your placement in this course.

Should you have any questions over the summer, send me an email! I am more than happy to help.

Parts of this packet include definitions and examples, these are here to help you. **You are only required to answer the exercise questions at the end of each section; however, you should understand EVERYTHING in the packet. Please show all work!**

This packet will take some time - please do not wait until the last minute! Ideally, spread the work out so you do not become overwhelmed.

Have a safe and restful summer, see you in August! 😃
Mrs. French

1.1 - Real Numbers, Functions, and Graphs

A real number is a number represented by a decimal or "decimal expansion." There are three types of decimal expansions: *finite, repeating, and infinite but non repeating*. For example:

$\frac{3}{8} = 0.375$	$\frac{1}{7} = 0.142857142857... = 0.\overline{142857}$	$\pi = 3.141592653589793....$

The set of all real numbers is denoted by a boldface **R.** We also use the standard symbol ϵ for the phrase "belongs to." Thus:

$a \in \mathbf{R}$ reads "a belongs to **R**"

The set of integers is commonly denoted by the letter **Z**.

Thus, $\mathbf{Z} = \{...., -2, -1, 0, 1, 2, ...\}$.

A whole number is a nonnegative integer - that is, 0, 1, 2, ….

A real number is called rational if it can be represented by a fraction $\frac{p}{q}$ where p and q are integers with $q \neq 0$. The set of rational numbers is denoted **Q**. Numbers that are not rational, such as π and $\sqrt{2}$, are called irrational.

We visualize real numbers as points on a line. For this reason, real numbers are often referred to as points. The point corresponding to 0 is called the origin.

The absolute value of a real number a, denoted $|a|$, is defined by:

$$|a| = \text{distance from the origin}$$

For example,

$$|1.2| = 1.2 \quad \text{and} \quad |-8.35| = 8.35$$

Also, absolute value satisfies the following properties:

$\lvert a\rvert = \lvert -a\rvert$	$\lvert ab\rvert = \lvert a\rvert\,\lvert b\rvert$

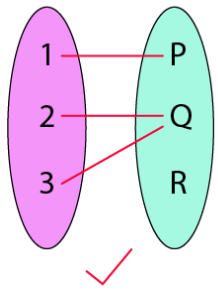
Please review the following examples of interval notation. Remember, we always use parentheses on infinity signs! We will use interval notation to describe domain & range.

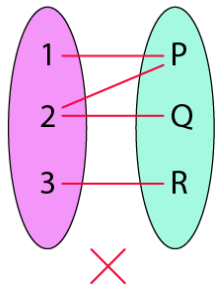
Intervals

Name of interval	Notation	Inequality description	Number line representation
Finite and closed	[a, b]	$a \le x \le b$	
Finite and open	(a, b)	$a < x < b$	
Finite and half-open	[a, b) (a, b]	$a \le x < b$ $a < x \le b$	 
Infinite and closed	$(-\infty, b]$ $[a, +\infty)$	$-\infty < x \le b$ $a \le x < +\infty$	 
Infinite and open	$(-\infty, b)$ $(a, +\infty)$	$-\infty < x < b$ $a < x < +\infty$	 
Infinite and open	$(-\infty, +\infty)$	$-\infty < x < +\infty$	

A function is a relation in which each input (x) is related to exactly one output (y). Informally, we think of the function f as a "machine" that produces an output y for every input x in the domain.

Domain = allowable inputs (x), independent variable
Range = outputs (y), dependent variable



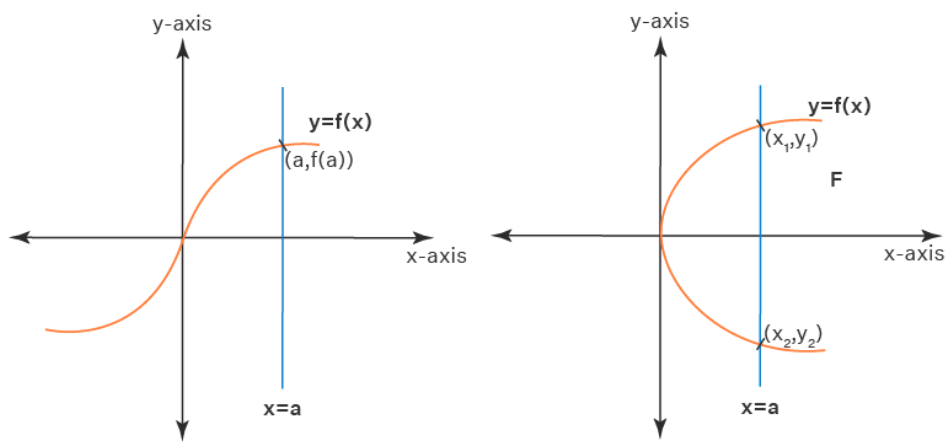


Graphs

A zero or root of a function f are the values of x where the graph intersects the x axis. To sketch a graph by hand, make a table of values, plot these points (including any zeros), and connect them with a smooth curve.

Remember - a curve is the graph of a function if and only if it passes the Vertical Line Test; that is, every vertical line intersects the curve in at most one point.

Vertical Line Test



Vertical Line Test- ✓
(It is a function)

Vertical Line Test- X
(Not a function)

We are often interested in whether a function is increasing or decreasing. Roughly speaking, a function f is increasing if its graph goes up as we move to the right, and is decreasing if its graph goes down.

Another important property is parity, which refers to whether a function is even or odd:

- f is even if $f(-x) = f(x)$
  - Graph is symmetric about the y axis
- f is odd if $f(-x) = -f(x)$
  - Graph is symmetric with respect to the origin

Two important ways of modifying a graph are translation (shifting) and scaling.

Translation (Shifting)

- Vertical Translation; $y = f(x) + c$; shifts the graph by c units vertically (upward if c > 0 and downward if c < 0)
- Horizontal Translation; $y = f(x + c)$; shifts the graph by c units horizontally to the right if c < 0 and c units to the left if c > 0.

Translation (Scaling)

- Vertical Scaling; $y = kf(x)$
  - If $k > 1$, the graph is expanded vertically by the factor of k.
  - If $0 < k < 1$, the graph is compressed vertically.
  - If $k < 0$, the graph is also reflected across the x-axis.
- Horizontal Scaling; $y = f(kx)$
  - If $k > 1$, the graph is compressed in the horizontal direction.
  - If $0 < k < 1$, the graph is expanded.
  - If $k < 0$, then the graph is also reflected across the y-axis.

Section 1.1 Practice Exercises

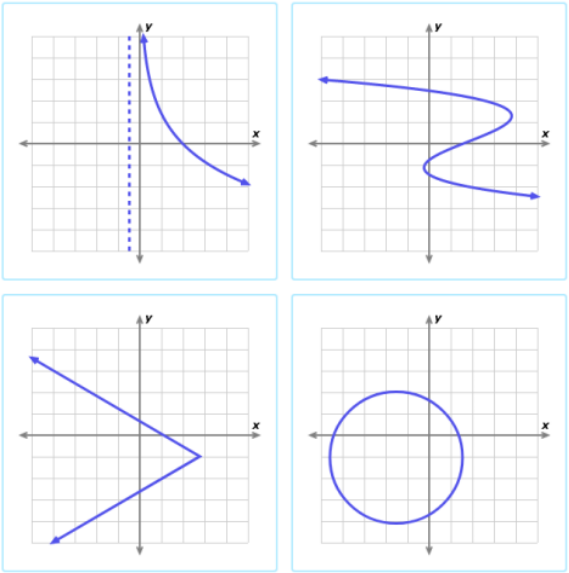
1) Find the domain and range of the following functions. The first is done for you as an example. Please use interval notation.

Function	Domain	Range
$f(x) = -x$	$(-\infty, \infty)$	$(-\infty, \infty)$
$f(x) = x^3$		
$f(x) = \lvert x \rvert$		
$f(x) = \frac{1}{x^2}$		

2) Determine whether the function is even, odd, or neither. Show all work.

$f(x) = x^5$	$f(x) = x^3 - x^2$	$f(x) = \frac{1}{x^4+x^2}$

3) Which of the following curves is the graph of a function? Circle your answer.



What did you use to determine your answer?

4) Find the function f whose graph is obtained by shifting the parabola $y = x^2$ by 3 units to the right and 4 units down.

5) For the following functions, determine where f is increasing. Please use interval notation.

a) $f(x) = |x + 1|$

b) $f(x) = x^4$

1.2 - Linear and Quadratic Functions

A linear function is a function of the form $f(x) = mx + b$ where m (the slope) and b (the y intercept) are constants.

Slope Intercept Form

$$y = mx + b$$

We calculate the slope (rate of change) with the following ratio:

$$m = \frac{\Delta y}{\Delta x} = \frac{vertical\ change}{horizontal\ change} = \frac{rise}{run}$$

Finding the Slope of a Line Algebraically:

How do we find the slope (denoted as m) of a line given two points (x_1, y_1) and (x_2, y_2)?

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example: Find the slope of the line containing the points $(-6, -1)$ and $(3, 2)$.

$$m = \frac{2 - (-1)}{3 - (-6)} = \frac{3}{9} = \frac{1}{3}$$

Note the following properties of linear functions!

- Positive Slope: If $m > 0$, the line slants upward from left to right.
- Negative Slope: If $m < 0$, the line slants downward from left to right.
- Horizontal Line: $m = 0$
- Vertical Line: m is undefined! Equation looks like $x = c$ where c is a constant. This is NOT a function!

Parallel & Perpendicular Lines

- Parallel lines have the SAME slope

$$y = 3x - 1$$
$$y = 3x + 5$$

- Perpendicular lines have OPPOSITE (NEGATIVE) RECIPROCAL slopes!

$$y = 2x + 4$$
$$y = -\frac{1}{2}x$$

Standard Form of a Linear Equation

$$ax + by = c$$

where a and b are not both zero.

Point-Slope Form of a Linear Equation

$$y - y_1 = m(x - x_1)$$

where the line passes through the point (x_1, y_1)

Writing an Equation of a Line Using Point-Slope Form:

What is point-slope form?

$y - y_1 = m(x - x_1)$

Example: Find the equation of the line containing the points $(-6, -1)$ and $(3, 2)$ using point-slope form.

Solution: First find the slope between the two points.

$$m = \frac{2-(-1)}{3-(-6)} = \frac{3}{9} = \frac{1}{3}$$

Now, write the equation of the line.

$$y - (-6) = \frac{1}{3}(x - (-1))$$

$$y + 6 = \frac{1}{3}(x + 1)$$

A quadratic function is a function defined by a quadratic polynomial

$$f(x) = ax^2 + bx + c$$

where a, b, and c are constants and $a \neq 0$

The graph of f is a parabola. The parabola opens upward if the coefficient of a is positive and downward if a is negative.

The roots of f are given by the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic functions may have two real roots, a repeated root, or no real roots. You can use the discriminant to determine how many roots a quadratic function has:

Discriminant: $D = b^2 - 4ac$

- If $D > 0$, then f has two real roots
- If $D = 0$, it has one real root (a "double" or repeated root)
- If $D < 0$, then $\sqrt{D}$ is imaginary and f has no real roots.

Section 1.2 Practice Exercises

1) Find the slope, y intercept, and x intercept of the line with the given equation:

   a) $y = 3x + 12$

   b) $y = 4 - x$

   c) $4x + 9y = 3$

2) Find the equation of the line with the given description:

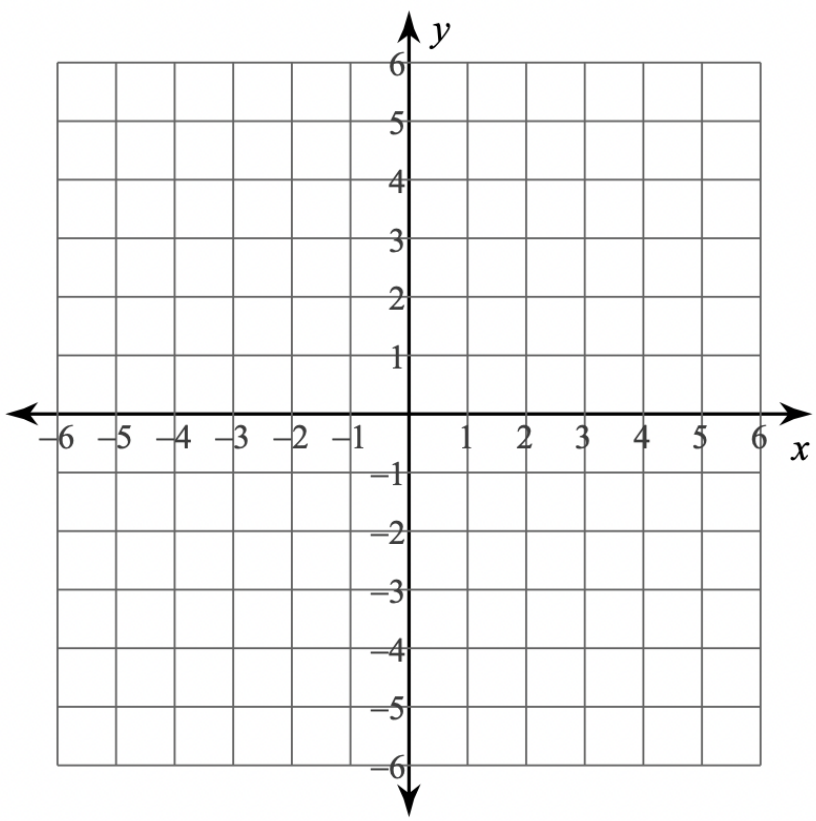
a) Vertical, passes through (-4, 9)	b) Slope 3, passes through (7, 9)
c) Passes through (1, 4) and (12, -3)	d) Perpendicular to $3x + 5y = 9$, passes through (2, 3)
e) Parallel to $y = 3x - 4$, passes through (1, 1)	

3) Find the roots of the quadratic polynomials:

a) $f(x) = 4x^2 - 3x - 1$	b) $f(x) = x^2 - 2x - 1$

4) Sketch the graph of $y = x^2 + 4x + 6$ by plotting the minimum point (vertex), the y intercept, and one other point. **<u>Do not use a graphing calculator!</u>**

***Hint* - find the *x* coordinate of the vertex by using the formula $x = \frac{-b}{2a}$**



1.3 - The Basic Classes of Functions

Most of the functions in this course are constructed from the following familiar classes of well-behaved functions. We will refer to these as the basic functions.

☆**Polynomials**: For any real number m, $f(x) = x^m$ is called the power function with exponent m. The base is a variable and the exponent is a constant. A polynomial is a sum of multiples of power functions with exponents that are positive integers or zero (making the term a constant in that case). Below are some examples:

$f(x) = x^5 - 5x^3 + 4x$	$h(x) = x^9$

☆**Rational Functions:** A rational function is a quotient of two polynomials: $f(x) = \frac{P(x)}{Q(x)}$ [$P(x)$ and $Q(x)$ are polynomials].

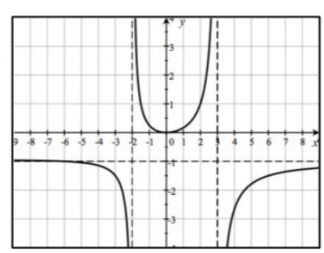
The domain of a rational function is the set of numbers x such that the denominator, $Q(x)$, does NOT equal 0. Below are a few examples:

$f(x) = \frac{1}{x^2}$ Domain: $\{x\colon x \neq 0\}$	$h(t) = \frac{7t^6+t^3-3t-1}{t^2-1}$ Domain: $\{t\colon t \neq \pm 1\}$

Rational functions in the form of $y = \frac{p(x)}{q(x)}$ possibly have vertical asymptotes, lines that the graph of the curve approach but never cross. To find the **vertical asymptotes**, factor out any common factors of numerator and denominator, reduce if possible, and then set the denominator equal to zero and solve.

Horizontal asymptotes are lines that the graph of the function approaches when x gets very large or very small. While you learn how to find these in calculus, a rule of thumb is that if the highest power of x is in the denominator, the horizontal asymptote is the line $y = 0$. If the highest power of x is both in numerator and denominator, the horizontal asymptote will be the line $y = \frac{\text{highest degree coefficient in numerator}}{\text{highest degree coefficient in denominator}}$. If the highest power of x is in the numerator, there is no horizontal asymptote, but a slant asymptote which is not used in calculus.

1) Find any vertical and horizontal asymptotes for the graph of $y = \frac{-x^2}{x^2-x-6}$.



$$y = \frac{-x^2}{x^2-x-6} = \frac{-x^2}{(x-3)(x+2)}$$

Vertical asymptotes: $x-3=0 \Rightarrow x=3$ and $x+2=0 \Rightarrow x=-2$
Horizontal asymptotes: Since the highest power of x is 2 in both numerator and denominator, there is a horizontal asymptote at $y=-1$.

☆**Exponential Functions:** the function $f(x) = b^x$, where $b > 0$, is called the exponential function with base b. Below are some examples:

$f(x) = 2^x$	$g(t) = 10^t$	$h(x) = (\frac{1}{3})^x$	$p(t) = (\sqrt{5})^t$

☆**Logarithmic Functions:** The inverse of an exponential function is a logarithmic function.

☆**Trigonometric Functions:** are functions built from $sinx$ and $cosx$. More to follow on this!

Constructing New Functions

Given functions f and g, we can construct new functions by forming the sum, difference, product, and quotient functions:

$(f + g)(x) = f(x) + g(x)$	$(f - g)(x) = f(x) - g(x)$
$(fg)(x) = f(x)g(x)$	$(\frac{f}{g})(x) = \frac{f(x)}{g(x)}$

For example, suppose $f(x) = x^2$ and $g(x) = sinx$, then:

$(f + g)(x) = x^2 + sinx$	$(f - g)(x) = x^2 - sinx$
$(fg)(x) = x^2 sinx$	$(\frac{f}{g})(x) = \frac{x^2}{sinx}$

Composition is another important way of constructing new functions. The composition of f and g is the function $f \circ g$ defined as $(f \circ g)(x) = f(g(x))$. Essentially, you are placing the function $g(x)$ inside the function $f(x)$.

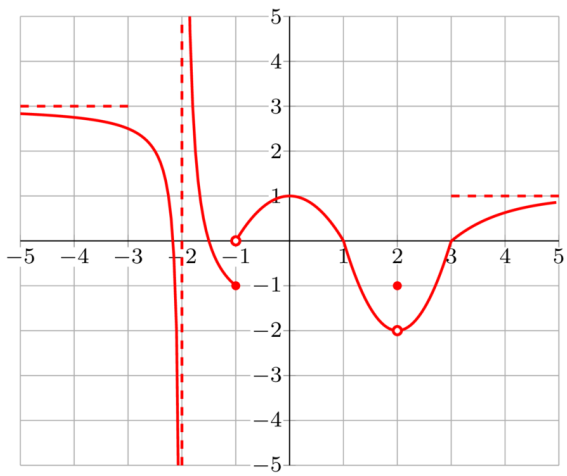
For example, suppose $f(x) = \sqrt{x}$ and $g(x) = 1 - x$, then we have

$$(f \circ g)(x) = f(g(x)) = f(1 - x) = \sqrt{1 - x}.$$

Finally, a piecewise function can be created by piecing together functions defined over limited domains. The absolute value function is actually a piecewise function! Below are a few examples:

$$|x| = \begin{cases} -x & \text{for } x < 0 \\ 0 & \text{for } x = 0 \\ x & \text{for } x > 0. \end{cases}$$

$$f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 6 & \text{if } x = 2 \\ 10 - x & \text{if } x > 2 \text{ and } x \leq 6 \end{cases}$$



Section 1.3 Practice Exercises

1. Determine the domain of the following functions. You may use set-builder notation or interval notation.

$f(x) = x^3 + 3x - 4$	$f(x) = \frac{1}{x+2}$	$f(x) = \frac{\sqrt{x}}{x^2-9}$

2. Calculate the functions $f \circ g$ and $g \circ f$.

$f(x) = \sqrt{x}$ $g(x) = x + 1$	$f(x) = \frac{1}{x}$ $g(x) = x^{-4}$	$f(x) = x^2$ $g(x) = cosx$

3. Answer the following questions about the piecewise function below..

$f(0) =$

$f(4) =$

$f(-1) =$

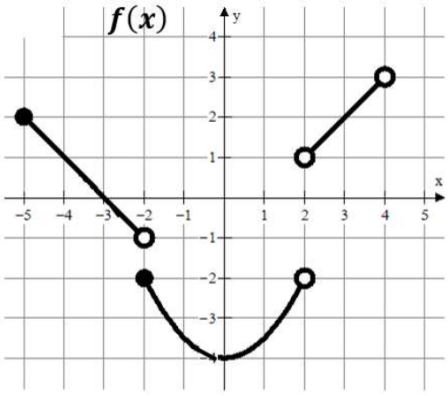
$f(-2) =$

$f(2) =$

$f(3) =$

$f(x) = 2$ when $x = ?$

$f(x) = -3$ when $x = ?$



4. Identify any vertical or horizontal asymptotes, and if present, the location of holes for the following graphs.

1. $y = \dfrac{x-1}{x+5}$

2. $y = \dfrac{8}{x^2}$

3. $y = \dfrac{2x+16}{x+8}$

4. $y = \dfrac{2x^2+6x}{x^2+5x+6}$

5. $y = \dfrac{x}{x^2-25}$

6. $y = \dfrac{x^2-5}{2x^2-12}$

1.4 - Trigonometric Functions

There are two systems of angle measurement; radians and degrees. They are best described using the relationship between angles and rotation. We use the lowercase Greek letter θ (theta) to denote angles and rotation.

The unit circle has a circumference 2π. Therefore, a rotation through a full circle has a radian measure $\theta = 2\pi$.

Rotation through	***Radian measure***
Two full circles	4π
Full circle	2π
Half circle	π
Quarter circle	$2\pi/4 = \pi/2$
One-sixth circle	$2\pi/6 = \pi/3$

Degrees are defined by dividing the circle into equal parts. A degree is 1/360 of a circle. A rotation through θ degrees is a rotation through the fraction $\theta/360$ of a complete circle.

To convert between radians and degrees, remember that 2π radians is equal to 360 degrees.

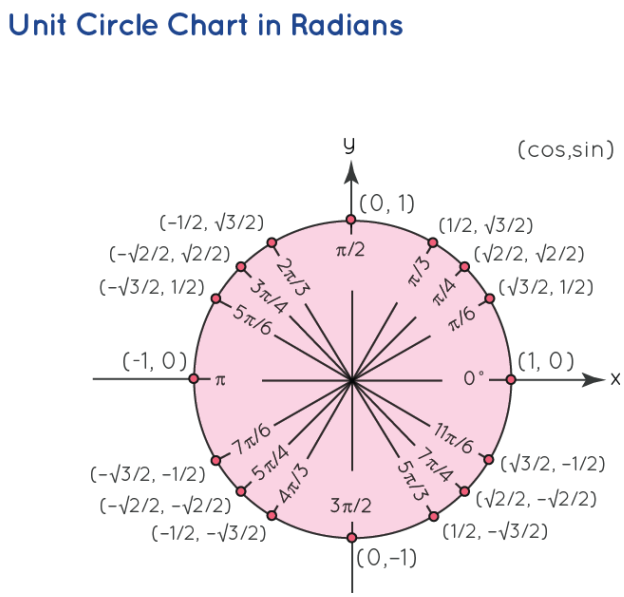
- To convert from radians to degrees, multiply by $180/\pi$
- To convert from degrees to radians, multiply by $\pi/180$

Radians	***Degrees***
0	0°
$\frac{\pi}{6}$	30°
$\frac{\pi}{4}$	45°
$\frac{\pi}{3}$	60°
$\frac{\pi}{2}$	90°

****Unless otherwise stated, we always measure angles in radians!****

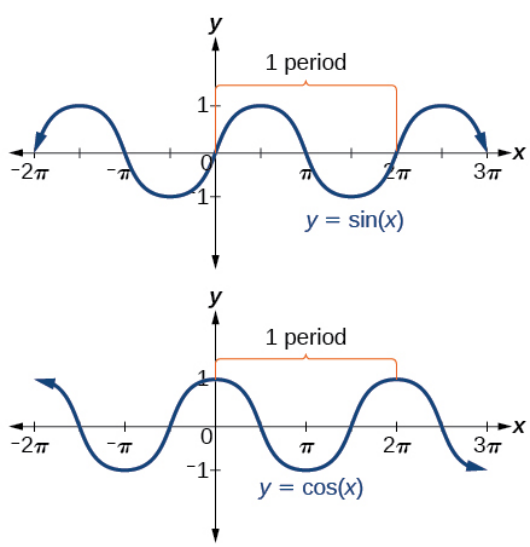
The Unit Circle

Although we can use a calculator to evaluate sine and cosine for general angles, the standard values listed in the image below appear often and should be memorized!



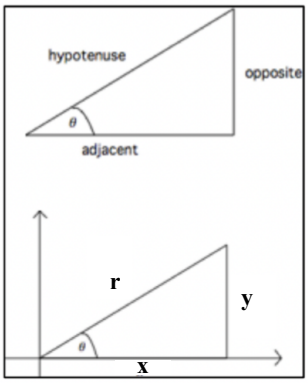
Graphs of Sine and Cosine

The graph of $y = sin\theta$ is the familiar "sine wave" shown below. The graph of $y = cos\theta$ has the same shape but is shifted to the left $\pi/2$ units. A function f is called periodic with period T if $f(x + T) = f(x)$. The sine and cosine functions are periodic with period $T = 2\pi$ because the radian measures x and $x + 2\pi k$ correspond to the same point on the unit circle for any integer k.



There are four other standard trigonometric functions, each defined in terms of $sinx$ and $cosx$ or as ratios of sides in a right triangle.

Given a right triangle with one of the angles named θ, and the sides of the triangle relative to θ named opposite (y), adjacent (x) , and hypotenuse (r) we define the 6 trig functions to be:



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{r} \qquad \csc\theta = \frac{\text{hyptotenuse}}{\text{opposite}} = \frac{r}{y}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{r} \qquad \sec\theta = \frac{\text{hyptotenuse}}{\text{adjacent}} = \frac{r}{x}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{x}{y}$$

The Pythagorean theorem ties these variables together: $x^2 + y^2 = r^2$. Students should recognize right triangles with integer sides: 3-4-5, 5-12-13, 8-15-17, 7-24-25. Also any multiples of these sides are also sides of a right triangle. Since r is the largest side of a right triangle, it can be shown that the range of $\sin\theta$ and $\cos\theta$ is $[-1,1]$, the range of $\csc\theta$ and $\sec\theta$ is $(-\infty,-1]\cup[1,\infty)$ and the range of $\tan\theta$ and $\cot\theta$ is $(-\infty,\infty)$.

Also vital to master is the signs of the trig functions in the four quadrants. A good way to remember this is A – S – T – C where All trig functions are positive in the 1st quadrant, Sin is positive in the 2nd quadrant, Tan is positive in the 3rd quadrant and Cos is positive in the 4th quadrant.

1. Let be a point on the terminal side of θ. Find the 6 trig functions of θ. (Answers need not be rationalized).

a) $P(-8,6)$

$x=-8, y=6,\ r=10$

$\sin\theta = \frac{3}{5}$ $\qquad \csc\theta = \frac{5}{3}$

$\cos\theta = -\frac{4}{5}$ $\qquad \sec\theta = -\frac{5}{4}$

$\tan\theta = -\frac{3}{4}$ $\qquad \cot\theta = -\frac{4}{3}$

b. $P(1,3)$

$x=1,\ y=3,\ r=\sqrt{10}$

$\sin\theta = \frac{3}{\sqrt{10}}$ $\qquad \csc\theta = \frac{\sqrt{10}}{3}$

$\cos\theta = \frac{1}{\sqrt{10}}$ $\qquad \sec\theta = \sqrt{10}$

$\tan\theta = 3$ $\qquad \cot\theta = \frac{1}{3}$

c. $P\left(-\sqrt{10},-\sqrt{6}\right)$

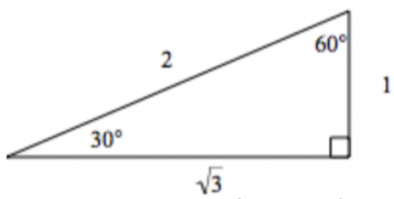
$x=-\sqrt{10},\ y=-\sqrt{6},\ r=4$

$\sin\theta = -\frac{\sqrt{6}}{4}$ $\qquad \csc\theta = -\frac{4}{\sqrt{6}}$

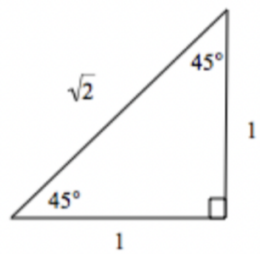
$\cos\theta = -\frac{\sqrt{10}}{4}$ $\qquad \sec\theta = -\frac{4}{\sqrt{10}}$

$\tan\theta = \sqrt{\frac{3}{5}}$ $\qquad \cot\theta = \sqrt{\frac{5}{3}}$

Because over half of the AP exam does not use a calculator, you must be able to determine trig functions of **special angles**. You must know the relationship of sides in both $30° - 60° - 90°$ $\left(\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}\right)$ and $45° - 45° - 90°$ $\left(\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}\right)$ triangles.



In a $30° - 60° - 90°$ $\left(\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}\right)$ triangle, the ratio of sides is $1 - \sqrt{3} - 2$.



In a $45° - 45° - 90°$ $\left(\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}\right)$ triangle, the ratio of sides is $1 - 1 - \sqrt{2}$.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$30°$ $\left(\text{or } \frac{\pi}{6}\right)$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$45°$ $\left(\text{or } \frac{\pi}{4}\right)$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$60°$ $\left(\text{or } \frac{\pi}{3}\right)$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Special angles are any multiple of $30°$ $\left(\frac{\pi}{6}\right)$ or $45°\left(\frac{\pi}{2}\right)$. To find trig functions of any of these angles, draw them and find the **reference angle** (the angle created with the *x*-axis). Although most problems in calculus will use radians, you might think easier using degrees. This will create one of the triangles above and trig functions can be found, remembering to include the sign based on the quadrant of the angle. Finally, if an angle is outside the range of $0°$ to $360°(0 \text{ to } 2\pi)$, you can always add or subtract $360°(2\pi)$ to find trig functions of that angle. These angles are called **co-terminal angles**. It should be pointed out that $390° \neq 30°$ but $\sin 390° = \sin 30°$.

• Find the exact value of the following

a. $4\sin 120° - 8\cos 570°$

Subtract $360°$ from $570°$
$4\sin 120° - 8\cos 210°$
$120°$ is in quadrant II with reference angle $60°$.
$210°$ is in quadrant III with reference angle $30°$.
$4\left(\frac{\sqrt{3}}{2}\right) - 8\left(\frac{-\sqrt{3}}{2}\right) = 6\sqrt{3}$

b. $\left(2\cos\pi - 5\tan\frac{7\pi}{4}\right)^2$

$(2\cos 180° - 5\tan 315°)^2$
$180°$ is a quadrant angle
$315°$ is in quadrant III with reference angle $45°$
$[2(-1) - 5(-1)]^2 = 9$

Trigonometric Identities

Trig identities are equalities involving trig functions that are true for all values of the occurring angles. While you are not asked these identities specifically in calculus, knowing them can make some problems easier. The following chart gives the major trig identities that you should know. To prove trig identities, you usually start with the more involved expression and use algebraic rules and the fundamental trig identities. A good technique is to change all trig functions to sines and cosines.

Fundamental Trig Identities
$\csc x = \frac{1}{\sin x}, \quad \sec x = \frac{1}{\cos x}, \quad \cot x = \frac{1}{\tan x}, \quad \tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{\cos x}{\sin x}$
$\sin^2 x + \cos^2 x = 1, \quad 1 + \tan^2 x = \sec^2 x, \quad 1 + \cot^2 x = \csc^2 x$
Sum Identities
$\sin(A+B) = \sin A \cos B + \cos A \sin B \qquad \cos(A+B) = \cos A \cos B - \sin A \sin B$
Double Angle Identities
$\sin(2x) = 2\sin x \cos x \qquad \cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$

Solving Trig Equations

Trig equations are equations using trig functions. Typically they have many (or infinite) number of solutions so usually they are solved within a specific domain. Without calculators, answers are either quadrant angles or special angles, and again, they must be expressed in radians.

For trig inequalities, set both numerator and denominator equal to zero and solve. Make a sign chart with all these values included and examine the sign of the expression in the intervals. Basic knowledge of the sine and cosine curve is invaluable from section R is invaluable.

• Solve for x on $[0, 2\pi)$

1. $x\cos x = 3\cos x$

Do not divide by $\cos x$ as you will lose solutions

$\cos x(x-3) = 0$

$\cos x = 0 \qquad x - 3 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = 3$

You must work in radians.

Saying $x = 90°$ makes no sense.

2. $\tan x + \sin^2 x = 2 - \cos^2 x$

$\tan x + \sin^2 x + \cos^2 x = 2$

$\tan x + 1 = 2$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

Two answers as tangent is positive in quadrants I and III.

Section 1.4 Practice Exercises

1. Convert from radians to degrees:

a) 1	b) $\frac{\pi}{3}$	c) $-\frac{3\pi}{4}$

Convert from degrees to radians:

d) 1°	e) 30°	f) 120°

2. If $\tan\theta = \frac{12}{5}$, θ in quadrant III, find $\sin\theta$ and $\cos\theta$

3. If $\csc\theta = \frac{6}{5}$, θ in quadrant II, find $\cos\theta$ and $\tan\theta$

4. $\cot\theta = \frac{-2\sqrt{10}}{3}$ find $\sin\theta$ and $\cos\theta$

Verify the following trig identities:

1. $(1+\sin x)(1-\sin x) = \cos^2 x$

2. $\sec^2 x + 3 = \tan^2 x + 4$

3. $\frac{1-\sec x}{1-\cos x} = -\sec x$

4. $\frac{1}{1+\tan x} + \frac{1}{1+\cot x} = 1$

Solve the following trig equations:

3. $3\tan^2 x - 1 = 0$

4. $3\cos x = 2\sin^2 x$

Essential Algebra Skills
PLEASE SHOW ALL WORK!

Factoring Polynomials:

Below is the thought process for factoring polynomials!

Step 1: Identify the GCF between the terms and factor it out if the GCF $\neq 1$. If the leading term (term with the highest power) is negative then you must also factor out a negative first! Then look inside the parentheses and see if it can factored further.

Step 2: If the GCF between all the terms is 1, then look to see how many terms you have!

Technique for 4 terms:

Use group factoring! Technique is explained below.

- Group the first two terms and the last two terms together.
- Then factor out the GCF from each of the two sets of parentheses.
- After you factor out the GCF's, looking at your new expression with two terms, factor out the GCF again which will be a binomial.

Techniques for 3 terms (trinomials): For trinomials of the form $\mathrm{ax}^2 + bx + c$

a = 1: (technique explained below)
Think of factors of the c-value that add up to b-value

$a \neq 1$: (technique explained below)

- Multiply the a and c value and find factors that somehow add/subtract to the middle term.
- Expand out the expression into 4 terms.
- Use group factoring method to factor the expression.

Techniques for 2 terms (binomials):

Difference of perfect squares
$a^2 - b^2 = (a - b)(a + b)$

Sum/Difference of perfect cubes

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Factor the polynomials below completely!

a) $2x^2 + 12x + 16$

b) $x^2 - 3x + 10$

c) $x^2 + 2x - 24$

d) $x^2 - 16$

e) $8x^3 + 27$

f) $2x^3 - 18x$

g) $x^3 - 4x^2 + 2x - 8$

h) $5x^3 - x^2 - 5x + 1$

i) $5x^2 - 3x - 2$

j) $3x^2 + 7x + 2$

Zero Product Property: This property states that if $ab = 0$ then either $a = 0$ or $b = 0$ (or both are zero).

Using the Zero Product Property to solve equations:

Example: Solve $(3x + 1)(x - 2) = 0$ for x.

$3x + 1 = 0$ or $x - 2 = 0$

$3x = -1$ or $x = 2$

$x = -\frac{1}{3}$ or $x = 2$

Solve the equations below for x.

a) $2x(x - 1) = 0$

b) $(2x + 5)(x - 4) = 0$

c) $x(3x + 4)(2x - 1) = 0$

d) $(x - 3)(x + \frac{1}{2}) = 0$

e) $(\frac{3}{2}x + 4)(2x - \frac{1}{3}) = 0$

f) $2(x - 10)(4x + 5) = 0$

Solving Quadratic Equations by Factoring:

Example 1: Solve $15x^2 + 5x = 0$ for x.

Factor the left hand side of the equation and use the zero product property to solve for x.

$5x(3x + 1) = 0$

$5x = 0$ or $3x + 1 = 0$

$x = 0$ or $x = -\frac{1}{3}$

Example 2: Solve $x^2 + 3x - 14 = -2x + 10$ for x.

Bring all the terms to one side of the equation. Then, factor the left hand side of the equation and use the zero product property to solve for x.

$x^2 + 5x - 24 = 0$

$(x + 8)(x - 3) = 0$

$x + 8 = 0$ or $x - 3 = 0$

$x = -8$ or $x = 3$

Solve the equations below for x by factoring.

a) $12x^2 + 8x = 0$

b) $x^2 + 5x + 4 = 0$

c) $3x^2 + 11x - 6 = -2$

d) $4x^2 + 3x - 11 = 3x - 2$

Exponent Rules:

- $x^m \cdot x^n = x^{m+n}$ Ex: $2x^2 \cdot 3x^5 = 6x^{2+5} = 6x^7$
- $\dfrac{x^m}{x^n} = x^{m-n}$ Ex: $\dfrac{8x^9}{2x^5} = 4x^{9-5} = 4x^4$
- $(x^m)^n = x^{mn}$ Ex: $(2x^3)^2 = 2^2x^{3\cdot 2} = 4x^6$
- $x^0 = 1$ where $x \neq 0$ Ex: $(9xyz)^0 = 1$
- $x^{-n} = \frac{1}{x^n}$ Ex: $x^{-3} = \dfrac{1}{x^3}$
- $\sqrt[n]{x^m} = x^{\frac{m}{n}}$ Ex: $\sqrt[4]{x^3} = x^{\frac{3}{4}}$

Remember that for the square root symbol the root number is 2 but it is never labeled.

Simplify the expressions and make sure to use positive exponents in your final answer.

a) $7x^6 \cdot 2x^4$

b) $\dfrac{6x^7y^8}{2x^3y}$

c) $-2(8xy)^0$

d) $(4x^4y^{-2})^3$

e) $(3ab^6)^2(-2a^2b^{-2})^3$

f) $\sqrt{\dfrac{x^2}{y^2}}$

Rewrite the following expressions using rational exponents.

Example: $\dfrac{1}{\sqrt[3]{x^2}} = \dfrac{1}{x^{\frac{2}{3}}} = x^{-\frac{2}{3}}$

a) $\sqrt[5]{x^3} + \sqrt{x}$

b) $\sqrt[3]{x+1}$

c) $\dfrac{1}{\sqrt[3]{x^5}}$

d) $\dfrac{1}{x^{10}} - \dfrac{3}{x}$

e) $\dfrac{1}{2x^6} + \dfrac{1}{4\sqrt{x}}$

f) $2\sqrt{x^7+4}$

Rewrite the following expressions using roots/positive exponents.

Example: $x^{-\frac{2}{3}} + 2x^{-7} = \dfrac{1}{x^{\frac{2}{3}}} + \dfrac{2}{x^7} = \dfrac{1}{\sqrt[3]{x^2}} + \dfrac{2}{x^7}$

a) $x^{-\frac{5}{2}} + x^{\frac{1}{4}}$

b) $x^{-8} + 6x^{-3}$

c) $(2x+1)^{-16}$

d) $\frac{3}{2}x^{-1}$

e) $(3x^5+10)^{-\frac{1}{2}}$

f) $\dfrac{1}{(3x)^{\frac{4}{5}}}$

Multiplying Binomials using FOIL Method:

FOIL stands for First Outer Inner Last!

Example: Multiply $(2x+1)(x+4)$

Solution: $(2x+1)(x+4) = 2x^2 + 8x + x + 4 = 2x^2 + 9x + 4$

Multiply the binomials below and please show work!

a) $(x+2)(x-3)$

b) $(3x-2)(2x+7)$

Special Formulas for Binomials:

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $(a+b)(a-b) = a^2 - b^2$

Expand out the expressions and simplify using the special formulas above for binomials!

Example: $(\sqrt{x} - \sqrt{y})(\sqrt{x} + \sqrt{y}) = (\sqrt{x})^2 - (\sqrt{y})^2 = x - y$

a) $(\sqrt{3a} - \sqrt{4b})(\sqrt{3a} + \sqrt{4b})$

b) $(x^2 + y^2)(x^2 - y^2)$

c) $(2x + 3y)^2$

d) $(\sqrt{x} - 2)^2$

Function Evaluation:

Example: Suppose $f(x) = 2x^2$, find $f(x+1)$.

Solution: $f(x+1) = 2(x+1)^2 = 2(x^2+2x+1) = 2x^2+4x+4$

Suppose $f(x) = x^2 + 5$. Compute the following and please show work!

a) $f(-2)$

b) $f(x^3)$

c) $f(x-3)$

d) $f(2x+1)$

e) $f(x+h)$

f) $\dfrac{f(5)-f(1)}{5-1}$

g) $\dfrac{f(x+h)-f(x)}{h}$

Important Properties:

$e^{\ln(x)} = x$ and $\ln(e^x) = x$

Also note that $\ln(e) = \ln(e^1) = 1$ and $\ln(1) = \ln(e^0) = 0$

Simplify the following expressions using the important properties above.

a) $e^{\ln(5)}$

b) $\ln(e^8)$

c) $2e^{\ln(3)} + \frac{1}{3}\ln(e)$

d) $-4\ln(e^{2x}) + \ln(1)$

Solving Exponential Equations:

Example 1: Solve $7^{2x+1} = 7^5$.

Solution: Make sure the exponential's have the same base. Then set the exponents equal and solve for x.

$2x + 1 = 5 \Rightarrow 2x = 4 \Rightarrow x = 2$

Example 2: Solve $e^x = 2$.

Solution: Make sure the exponential is isolated. Then take the natural log of both sides and solve for x.

$e^x = 2 \Rightarrow \ln(e^x) = \ln(2) \Rightarrow x = \ln(2)$

Solve the equations below for x and show your work.

a) $12^{x+5} = 144$

b) $4^{3x-5} = 4^{x+7}$

c) $5^{2x-1} = \sqrt{5}$

d) $e^x + 1 = 4$

e) $3e^x + 5 = 8$

f) $3e^{2x} = 9$

g) $4^x + 5 = 8$

h) $\dfrac{e^x + 1}{2} = 7$

i) $2^{3x+1} = 4^x$

Solving Logarithmic Equations:

How do logarithmic functions work?

For $x > 0$, $b > 0$, and $b \neq 1$, we have that

$$\log_b(x) = y \Rightarrow b^y = x$$

Make sure you isolate the logarithm is the equation first before using the exponential form above!

Example: Solve the equation $\log_2(4x - 12) - 1 = 2$ for x.

Solution: First isolate the logarithm by adding 1 to both sides in this case. Then use its equivalent exponential form to solve for x.

$\log_2(4x - 12) = 3$

$2^3 = 4x - 12 \Rightarrow 8 = 4x - 12 \Rightarrow 4x = 20 \Rightarrow x = 5$

Solve the equations below for x and show your work! Recall that $\ln(x) = \log_e(x)$ and $\log(x) = \log_{10}(x)$

a) $\log_2(x) = 4$

b) $\log_3(x) = -3$

c) $\log_{64}(x) = \frac{1}{2}$

d) $\log_6(2x - 4) = 2$

e) $3\log_2(x + 4) - 5 = 10$

f) $2\ln(x) + 3 = 4$

g) $3\log(3x - 2) = 3$